NASA TT F-10,643

PROTECTIVE COOLING BY A MULTISLOT AND NETWORK-TYPE INJECTION

INTO A TURBULENT BOUNDARY LAYER

E. P. Volchkov, S. S. Kutateladze,
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NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON   DECEMBER 1966

PROTECTIVE COOLING BY A MULTISLOT AND NETWORK-TYPE INJECTION INTO A TURBULENT BOUNDARY LAYER

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ABSTRACT

The authors consider the effectiveness of
a protective cooling of an insulated plane sur-
face by gas injection into the turbulent boundary
layer of the incident flow through a system of
slots. An analytical method is developed to
assess the results of such a cooling technique,
taking the dimensionless temperature of the sur-
face as the measure of its effectiveness.

An analytical method is proposed for the calculation of the effective-   /149*
ness of a protective cooling of a plane heat-insulated wall by multislot
and network-type injection of a cooling gas. The dimensionless temperature
of the heat-insulated wall is taken to be the measure of the effectiveness
of the heat protection.

Apparently, the picture of flow in the boundary layer for the cases
examined appears to be more complex than during injection of a coolant
through a single slot. However, it seems feasible to extend to the cases
examined the calculating procedures proposed in the works [1,2].

Let us investigate a homogeneous turbulent boundary layer of gas with
constant physical properties in a prescribed range of temperatures. A
stream of gas with the temperature T_0 (°K) and the velocity w_q (m/sec) flows
around the surface (Fig. 1,a). The cooling gas is injected through several
sequential slots of a width s_1, ..., s_n (m) with a temperature T_1, ..., T_n,
and a velocity w_1, ..., w_n, respectively. Immediately behind the cross section
of each slot there is a region x_1, ..., x_n, in which the wall temperature is
constant and is equal to the temperature of the injected gas. The length of
this region may be found in the first approximation by utilizing the well-
known formulas for submerged flows [3].

There is no heat exchange through the wall, and the wall temperature

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*Numbers given in the margin indicate pagination in original foreign text

is a function of the x coordinate. The wall temperature behind the first slot may be found by calculating single-slot cooling, as, for example, in the works [1,2]. The problem consists of determining T_w^* behind the second, third, and so forth slots. In order to achieve this, it is necessary to determine the characteristic parameters of the boundary layers in the cross section of each slot.

1. The thickness of the energy loss in the cross section of the second slot may be expressed as follows:

$$\delta_{r2}^{**} = \int_0^\infty \frac{\rho w}{\rho_0 w_0} \left(\frac{T_0 - T}{T_0 - T_1}\right) dy = \int_0^{s_0} \frac{\rho w}{\rho_0 w_0} \left(\frac{T_0 - T}{T_0 - T_1}\right) dy + \int_{s_0}^\infty \frac{\rho w}{\rho_0 w_0} \left(\frac{T_0 - T}{T_0 - T_1}\right) dy = $$
$$= m_2 s_2 + \frac{T_0 - T_{w2}^*}{T_0 - T_1}(\delta_{r_0}^{**})_2 \qquad \left(m_2 = \frac{\rho_2 w_2}{\rho_0 w_0}\right)$$

(1.1)

where m_2 is the injection parameter of the second slot, and T_w2^* is the wall temperature over the second slot.

The integral

$$(\delta_{r_0}^{**})_2 = \int_{s_0}^\infty \frac{\rho w}{\rho_0 w_0} \left(\frac{T_0 - T}{T_0 - T_{w2}^*}\right) dy$$

represents the thickness of the energy loss of the boundary layer over the slot in the cross section of the second slot. Integrating the equation of energy of the boundary layer from x = 0 (the cross section of the first slot) to x = d (the cross section of the second slot); with $q_w = 0$

$$\frac{d\delta_r^{**}}{dx} + \frac{\delta_r^{**}}{\Delta T}\frac{d(\Delta T)}{dx} = 0$$

(1.2)

we find that the effectiveness of the protection of the wall over the second slot is equal to

$$\frac{T_0 - T_{w2}^*}{T_0 - T_1} = \frac{\delta_{r1}^{**}}{(\delta_{r_0}^{**})_2}$$
$$(\delta_{r_1}^{**} = m_1 s_1)$$

(1.3)

Here δ_{T1}^{**} is the thickness of the energy loss in the cross section of the first slot [1,2]. Consequently, it follows from the equalities (1.1) and (1.3) that

$$\delta_{r2}^{**} = m_2 s_2 + \frac{T_0 - T_1}{T_0 - T_2}\delta_{r1}^{**} = m_2 s_2 + \frac{T_0 - T_1}{T_0 - T_2} m_1 s_1$$

(1.4)

Similarly, it may be shown that the thickness of the energy loss in the <u>/150</u> cross section of the third slot is expressed as follows (in this case, the equation of the energy of the boundary layer (1.2) is integrated in the segment between the second and the third slots):

$$\delta_{r3}^{**} = m_3 s_3 + \frac{T_0 - T_2}{T_0 - T_3}\delta_{r2}^{**} = m_3 s_3 + \frac{T_0 - T_2}{T_0 - T_3} m_2 s_2 + \frac{T_0 - T_1}{T_0 - T_3} m_1 s_1$$

(1.5)

Finally, in the cross section of the n-th slot

$$\delta_{rn}^{**} = m_n s_n + \frac{T_0 - T_{n-1}}{T_0 - T_n} \delta_{T,n-1}^{**} \qquad (1.6)$$

or
$$\delta_{rn}^{**} = m_n s_n + \frac{T_0 - T_{n-1}}{T_0 - T_n} m_{n-1} s_{n-1} + \frac{T_0 - T_{n-2}}{T_0 - T_n} m_{n-2} s_{n-2} + \ldots + \frac{T_0 - T_1}{T_0 - T_n} m_1 s_1 \qquad (1.7)$$

2. The thickness of the loss of momentum is determined from the solution to the equation for momentum of the boundary layer, which for nongradient flow, has the form

$$\frac{dR^{**}}{dR_x} = \frac{C_f}{2} \qquad \left(R^{**} = \frac{\rho_0 w_0 \delta^{**}}{\mu_0}, \; R_x = \frac{\rho w_0 x}{\mu_0}, \; \delta^{**} = \int_0^\infty \frac{\rho w}{\rho_0 w_0} \left(1 - \frac{w}{w_0} \right) dy \right) \qquad (2.1)$$

Here R^{**} is the Reynolds number, constructed in relation to the thickness of the momentum loss; R_x is the Reynolds number, constructed in relation to the longitudinal coordinate; C_f if the local value of the friction coefficient; and δ^{**} is the thickness of the momentum loss. It is assumed, the same as in the works [1,2], that the boundary layer on the wall developed into turbulent flow with a power velocity profile. Here, the friction law is valid in the form

$$\tfrac{1}{2} C_f = A R^{**-a} \qquad (2.2)$$

Integrating Eq. (2.1) and taking Eq. (2.2) into account on the segment between the first and the second slot, we obtain

$$R^{**} = [R_1^{**(a+1)} + A (a+1) R_x]^{\frac{1}{a+1}} \qquad (2.3)$$

For the power velocity profile with the indicator $n = 1/7$, the calculations yield the values: $A = 0.0128$, $a = 0.25$. After transformation of the equality (2.3) we have

$$\Delta = [1 + 0.016 \chi^{1.25}]^{0.8} \qquad \left(\Delta = \frac{\delta^{**}}{\delta_1^{**}}, \quad \chi = \frac{x}{\delta_1^{**} R_x^{0.2}} \right) \qquad (2.4)$$

Here δ_1^{**} is the thickness of the momentum loss in the cross section of the first slot. The relation $\Delta = \Delta (\chi)$, constructed in Figure 2 according to formula (2.4), is satisfactorily verified by the results of experiments [4,5]. The thickness of the momentum losses of the boundary layer over the slot in the cross section of the second slot from the equality (2.4) is

$$(\delta_0^{**})_2 = \left[\delta_1^{**1.25} + 0.016 \left(\frac{d}{R_d^{0.2}} \right)^{1.25} \right]^{0.8} \qquad (2.5)$$
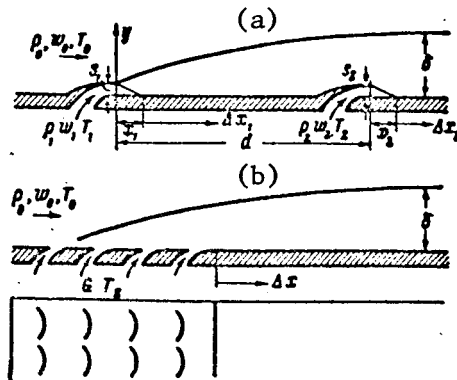
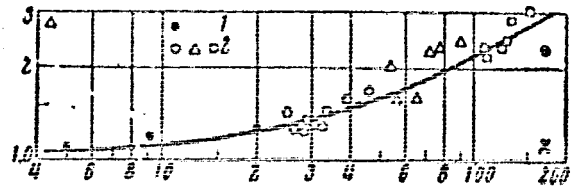Figure 1. Diagrams of the multislot (a) and network-type (b) cooling.

Figure 2. Variation of the thickness of momentum losses during the injection through a tangetial slot ($w/sw_0 \ll 1$); points are: 1-experiments [4]; 2-experiments [5].

The total thickness of the momentum loss in this cross section, taking injection through the slot into account, is

$$\delta_2^{**} = m_2 s_2 \left(1 - \frac{w_2}{w_0}\right) + (\delta_0^{**})_2 \qquad (2.6)$$

The thickness of the momentum loss in the cross section of the n-th slot may be found in a similar manner:

$$\ell_n^{**} = m_n s_n \left(1 - \frac{w_n}{w_0}\right) + (\delta_0^{**})_n \qquad (2.7)$$

Here $(\delta_0^{**})_n$ is found by progressive integration of the momentum equation in the form of (2.1), taking into account the friction law (2.2) on the segments between the slots.

The local friction coefficient behind the n-th slot from Eqs. (2.1) and (2.2) with the boundary condition (2.7) is

$$\frac{C_f}{2} = \frac{0.0128}{[R_n^{**1.25} + 0.010 R_{x_n}]^{0.2}} \qquad (2.8)$$

Here x_n is the distance counted off from the cross section of the n-th slot.

3. The effectiveness of the heat protection is investigated. It may be seen from Figure 2 that the local value of the thickness of momentum loss differs substantially from the value in the cross section of the slot only at

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large distances.

It was shown [6,7] that local variations in the dynamic field of the flow have only a secondary effect on the heat transfer process.
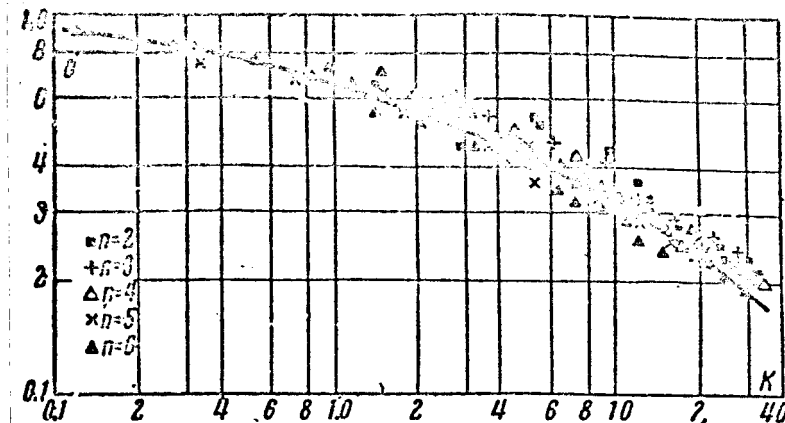

Figure 3. Curve according to formula (3.6);
points are: experiments [3] with $v_s/w_*< 0.2, 0 < d/s < 78$

In view of the above, an analysis of the effectiveness of the heat protection in the first approximation may include the assumption that the momentum variation to the n-th slot proceeds only at the expense of the injection through the slots (i.e., friction on the wall between the slots to the n-th slot is ignored).
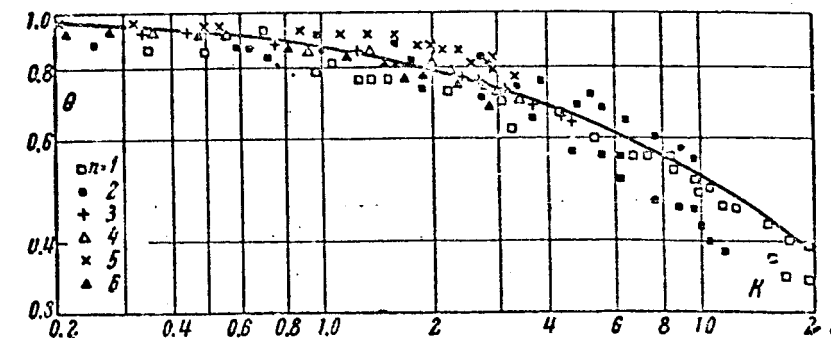

Figure 4. Curve according to formula (3.7);
points are: experiments [8] with $0.615 < v_s/v_* < 1.53$;
$0 < d/s < 78$

Then the expression for the thickness of momentum loss in the cross section of the n-th slot is simplified and may be written in the form

$$\delta_n^{**} = \int_0^{\infty} \frac{\rho_1 w_1}{\rho_0 w_0}\left(1 - \frac{w_1}{w_0}\right) dy + \int_0^{\infty} \frac{\rho_2 w_2}{\rho_0 w_0}\left(1 - \frac{w_2}{w_0}\right) dy + \ldots$$

$$\ldots + \int_0^{\infty} \frac{\rho_n w_n}{\rho_0 w_0}\left(1 - \frac{w_n}{w_0}\right) dy = m_1 s_1\left(1 - \frac{w_1}{w_0}\right) + m_2 s_2\left(1 - \frac{w_2}{w_0}\right) + \ldots + m_n s_n\left(1 - \frac{w_n}{w_0}\right) \quad (3.1)$$

The formulas for the calculation of the effectiveness of the heat protection of a heat-insulated wall during the injection through several sequential slots may be obtained by the employment of the procedures proposed for the case of injection of a coolant through a single slot [1,2]. The difference consists only in the determination of the initial parameters of the boundary layer; these are calculated in the cross section of the n-th slot taking into account the injection of the coolant through all the preceding slots according to the equalities (1.7) and (3.1). These quantities are taken into account as boundary conditions during the integration of the equation of energy (1.2) and momentum (2.1) of the boundary layer on the wall behind the n-th slot.

Employing the procedures described in the literature [2], it may be /152 shown that the formulas obtained in that work for the effectiveness of the heat covering may be extended to the case of a multislot injection of the coolant.
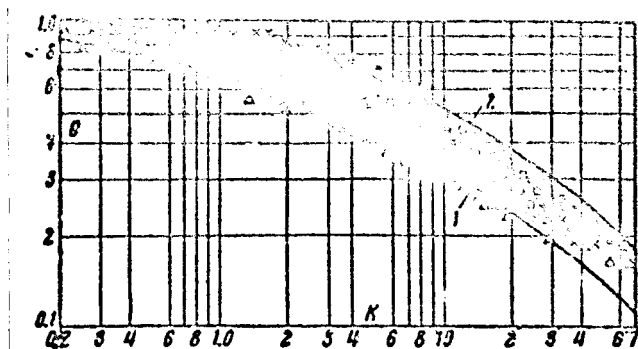

Figure 5. Generalization of the data on cooling of a heat-insulated wall during multislot injection; curves 1 and 2 are given according to formulas (3.6) and (3.7). The experimental points [8] with $0 < w_s/w_s < 1.33$, have the same designations as those in Figures 3 and 4.

According to the work [2], for the power velocity profile with n = 1/7,

we obtain

$$\Theta = \left[\left(\frac{R_{rx}^{\bullet\bullet}}{R_{r\Delta x}^{\bullet\bullet}}\right)^{0.25}\left(\frac{R_{\Delta x}^{\bullet\bullet}}{R_x^{\bullet\bullet}}\right)^{0.107} - 1\right]^{0.8}\left(\frac{R_{rn}^{\bullet\bullet}}{R_{rx}^{\bullet\bullet}}\right)^{0.2} \tag{3.2}$$

$$\Theta = \frac{T_0 - T_w^*}{T_0 - T_n^*}, \qquad \begin{aligned} R_{rx}^{\bullet\bullet} &= [R_{rn}^{\bullet\bullet 1.25} + 0.016\,R_{\Delta xn}]^{0.8} \\ R_x^{\bullet\bullet} &= [R_n^{\bullet\bullet 1.25} + 0.016\,R_{\Delta xn}]^{0.8} \\ R_{r\Delta x}^{\bullet\bullet} &= R_{\Delta x}^{\bullet\bullet} = [0.016\,R_{\Delta xn}]^{0.8} \end{aligned}$$

Henceforth, we shall assume for the sake of simplicity that

$$w_s = w_1 = \ldots = w_n, \qquad T_s = T_1 = \ldots = T_n, \qquad s_1 = \ldots = s_n \tag{3.3}$$

Then, from (1.7) and (3.1) we have

$$\delta_{rn}^{\bullet\bullet} = nms, \qquad \delta_n^{\bullet\bullet} = nms\left(1 - \frac{w_s}{w_0}\right) \tag{3.4}$$

Under these conditions, the equality (3.2) may be transformed to

$$\Theta = \frac{1}{(1 + 0.016\,K)^{0.16}}\left\{(1 + 62.5\,K^{-1})^{0.2}\left[1 + 62.5\left(1 - \frac{w_s}{w_0}\right)^{1.25}K^{-1}\right]^{-0.096} - 1\right\}^{0.8} \tag{3.5}$$

$$\Theta = \frac{T - T_w^*}{T_0 - T_s}, \qquad K = \frac{R_{\Delta rn}}{R_{rn}^{\bullet\bullet 1.25}} = \frac{R_{\Delta rn}}{R_{ns}^{1.25}}, \qquad R_{ns} = \frac{\rho_s w_s n_s}{\mu_0}$$

From expression (3.5) the following interpolation formulas are obtained for limiting cases:

$$\Theta = \left[\left(1 + \frac{62.5}{K + 0.143}\right)^{0.114} - 1\right]^{0.8}(1 + 0.016K)^{-0.16}\text{ with }\frac{w_s}{w_0} \ll 1 \tag{3.6}$$

$$\Theta = \left[\left(1 + \frac{62.5}{K + 2}\right)^{0.2} - 1\right]^{0.8}(1 + 0.016\,K)^{-0.16}\text{ with }\frac{w_s}{w_0} \approx 1 \tag{3.7}$$

A comparison of the relation $\Theta = \Theta\,(K)$, constructed according to formulas (3.6) and (3.7), with the results of experiment [8] is given in Figures 3-5.

In the experiments [5] on multislot and network injection of a coolant, the main stream already over the first slot had a certain dynamic and thermal boundary layer (i.e., there was an initial thickness of energy loss due to the cooling of the gas in the boundary layer before the working segment). Therefore, during the comparisons with the calculations only those experiments were taken in which the thickness of the energy loss due to the injection through the slots greatly exceeded the initial thickness of energy loss due to the cooling of the main flow through the wall before the first slot (i.e., experiments in which nms > 1 mm).

It may be seen from Figures 3-5 that the calculations are verified by

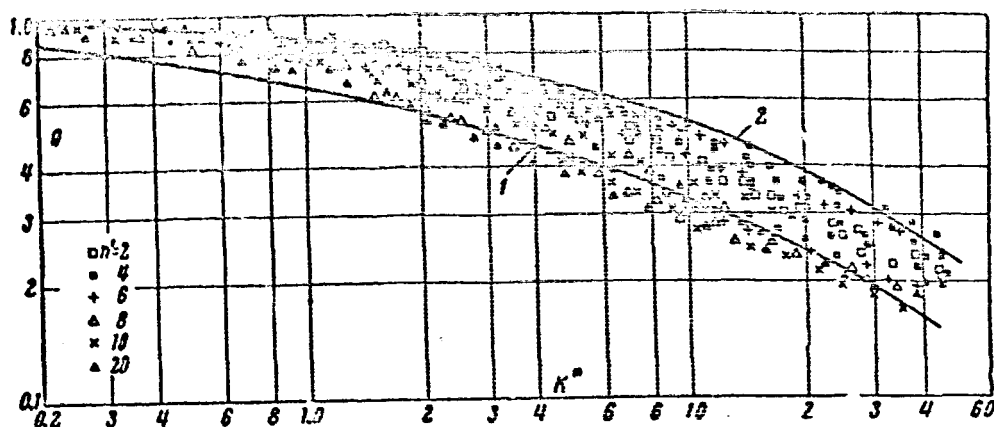the experimental data on the multislot cooling of a heat insulated wall.

Figure 6. Effectiveness of the heat protection
Θ as a function of K* during the injection of
a coolant through a tangential network with
$0 < w_s/w_0 < 1$; the curves 1 and 2 represent
calculations according to formulas (3.6) and
(3.7); the points are experimental [3]; here
h' is the number of open cut rows.)

Experimental data were also obtained [8] during the injection of a coolant through a network-like panel in which tangential slots had been made (Fig. 1,b).

In this case, the parameter K in the formulas (3.5) to (3.7) is reduced to the following formula which is convenient for practical application:

$$K = (\mu_s/G)^{1.25} R\Delta_s$$

Here G is the coolant flow rate per unit of surface width.

When the flow is expressed in that form, there is no need to determine slots equivalent in dimension, as it was done in the work [8]. As it is shown in Figure 6, even in such apparently, complex cases, the calculations are verified by experimental data [8].

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